DEPENDENCE OF THE TEMPERATURE COEFFICIENT OF DRYING ON THE HEAT- AND MASS-TRANSFER SIMILARITY CRITERIA FOR VARIOUS VALUES OF THE HEAT-TRANSFER BIOT NUMBER

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The influence of similarity criteria on the temperature coefficient of drying is analyzed for various values of the heat-transfer Biot number.

An experimental investigation of the influence of thickness of the material on the Rebinder number in the process of convective drying has revealed a difference in the behavior of the temperature coefficient b of drying for "thin" and "thick" samples [1].

The heat-transfer Biot number Bi_q for samples of various thicknesses has also been calculated in the processing of experimental data on the drying kinetics of porous ceramic, clay, and wood (pine). The calculations show that for samples with a thickness of 5 to 20 mm the number Bi_q assumes values less than unity ($Bi_q \approx 0.2$ to 0.3), while for a thickness of 30 to 60 mm it turns out to be greater than unity in every case ($Bi_q \approx 1.2$ to 5).

On the basis of our experimental data on the drying kinetics of samples of various thicknesses we have attempted to segregate "thin" and "thick" materials according to a combination of the values of the Rebinder number (or temperature coefficient of drying) and the heat-transfer Biot number.

It is worthwhile in this connection to use the solution of the system of differential equations of heat and mass transfer subject to boundary conditions of the third kind, making it possible to substantiate the influence of various conditions and parameters on the drying process and to set up experiments on a rigorous scientific foundation.

The differential equations of heat and moisture transfer [2, 3] for the one-dimensional problem have the form

$$\frac{\partial T(X,\ \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2 T(X,\ \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X} \cdot \frac{\partial T(X,\ \mathrm{Fo})}{\partial X} - \varepsilon \cdot \mathrm{Ko} \frac{\partial U(X,\ \mathrm{Fo})}{\partial \mathrm{Fo}}, \tag{1}$$

$$\frac{\partial U(X,\ \mathrm{Fo})}{\partial \mathrm{Fo}} = \mathrm{Ly}\left[\frac{\partial^2 U(X,\ \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X} \cdot \frac{\partial U(X,\ \mathrm{Fo})}{\partial X}\right] \mathrm{Ly}\,\mathrm{Pn}\left[\frac{\partial^2 T(X,\ \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X} \cdot \frac{\partial T(X,\ \mathrm{Fo})}{\partial X}\right]. \tag{2}$$

The dimensionless mass flux Ki_m for convective heat and mass transfer in the period of declining drying rate can be expressed by the relations

$$\mathrm{Ki}_m = \mathrm{Bi}_m [1 - U(1,\ \mathrm{Fo})], \tag{3}$$

$$\mathrm{Ki}_m = \mathrm{Ki}_m(\mathrm{Fo}); \tag{4}$$

i.e., the mass flux at the surface of the body is a function of the mass-transfer potential and is an implicit function of the time.

We have used a relation of the type (3) in an earlier investigation [4] of the influence of the heat- and mass-transfer similarity criteria on the temperature coefficient b on drying. In the period of declining drying rate, however, the dimensionless mass flux can often be expressed by a relation of the type (4) as well. Moreover, in solving the system of equations

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TABLE 1. Numerical Values of Heat-and Mass-Transfer Similarity Criteria as a Function of Heat-Transfer Biot Number

Investigated criterion		Values used for similarity criteria							
		ε KoPn	ε	Ko	Pn	Ly	ε Ko	K_2	Bi_q
Bi_q	0,1—1	0,1	0,416	1,2	0,2	0,3	0,5	2	Bi_q
	2—50	0,6	0,8	1,5	0,5	0,3	1,2	2	Bi_q
Ly	0,1—2	0,3	0,5	3	0,3	Ly	1,5	2	0,5
	0,1—2	0,3	0,6	1	0,5	Ly	0,6	2	10
ε	0,1—1	0,1—0,6	ε	3	0,2	0,3	0,5—3	2	0,5
	0,1—1	0,1—0,6	ε	1,2	0,5	0,3	0,2—2	2	10
K_2	0,1—5	0,1	0,416	1,2	0,2	0,3	0,5	K_2	0,5
	0,1—2	0,6	0,8	1,5	0,5	0,3	1,2	K_2	10

(1)-(2) an approximate expression is used for the drying-rate curve in the declining-rate period:

$$-\frac{d\bar{u}}{d\tau} = \varkappa N(\bar{u} - u_p), \tag{5}$$

which is well justified for diverse drying conditions.

An exact solution of the differential equations (1) and (2) for constant initial conditions and conditions (4) and (5) has been obtained by Mikhailov [3]:

$$T(X,\ \mathrm{Fo}) = 1 - \sum_{n=1}^{\infty}\sum_{i=1}^{2} C_{ni}\cos(\nu_i\mu_n X)\exp(-\mu_n^2\,\mathrm{Fo}); \tag{6}$$

$$U(X,\ \mathrm{Fo}) = 1 + \frac{1}{\varepsilon\,\mathrm{Ko}}\sum_{n=1}^{\infty}\sum_{i=1}^{2} C_{ni}(1-\nu_i^2)\cos(\nu_i\mu_n X)\exp(-\mu_n^2\,\mathrm{Fo}). \tag{7}$$

The coefficients C_{ni} [2, 3] depend on the numbers ε, Ko, Bi_q, Bi_m, Ly, Pn and differ appreciably, as shown by computations, from the corresponding coefficients in our earlier work [4].

The dimensionless warmup and drying rates are obtained by differentiating Eqs. (6) and (7) with respect to Fo:

$$\frac{\partial T}{\partial\,\mathrm{Fo}} = \sum_{n=1}^{\infty}\sum_{i=1}^{2} C_{ni}\frac{\mu_n}{\nu_i}\sin\nu_i\mu_n\exp(-\mu_n^2\,\mathrm{Fo}), \tag{8}$$

$$\frac{dU}{d\,\mathrm{Fo}} = \frac{1}{\varepsilon\,\mathrm{Ko}}\sum_{n=1}^{\infty}\sum_{i=1}^{2} C_{ni}\frac{\mu_n(1-\nu_i^2)}{\nu_i}\sin\nu_i\mu_n\exp(-\mu_n^2\,\mathrm{Fo}). \tag{9}$$

From expressions (8) and (9) we determine the temperature coefficient of drying:

$$b = \frac{dT/d\,\mathrm{Fo}}{dU/d\,\mathrm{Fo}}. \tag{10}$$

The solution of the system of differential equations gives the temperature coefficient of drying as a function of a large group of heat- and mass-transfer similarity criteria. For example, we can write

$$b = f(\mathrm{Fo},\ \mathrm{Ly},\ \mathrm{Bi}_q,\ \mathrm{Bi}_m,\ \varepsilon,\ \mathrm{Pn},\ \mathrm{Ko},\ \mathrm{Ki}_m,\ \text{etc.}).$$

However, not all the criteria affect the drying process in equal measure. The similarity criteria Ly, Bi_q, Ko, ε, Fo, and Pn have the most pronounced effect on convective drying. Besides these criteria, two additional dimensional parameters essential to convective drying are used in solving the equations:

$$K_1 = \frac{1-\varepsilon}{\varepsilon}\,\mathrm{Ly}\,\frac{K_2}{\mathrm{Bi}_q}, \tag{11}$$

$$K_2 = \frac{R_v \cdot R\varkappa N}{a_m}. \tag{12}$$

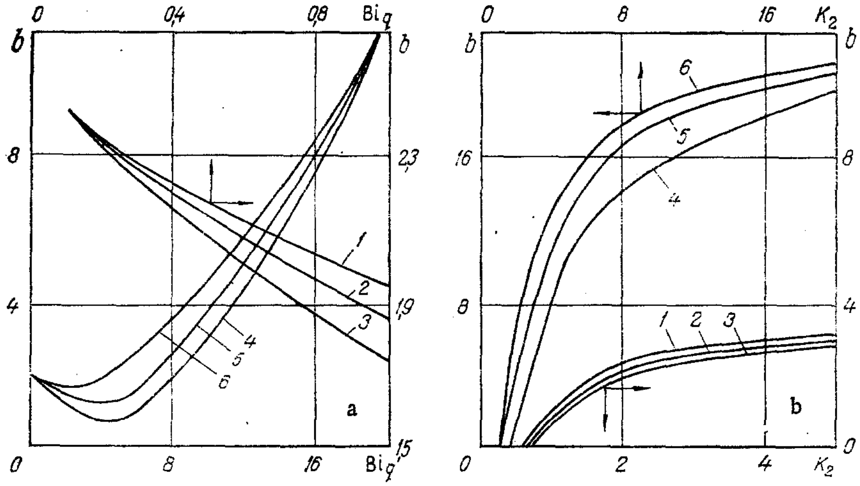

Fig. 1. Temperature coefficient of drying versus the number Bi_q (a) and the parameter K_2 (b). 1, 4) Fo = 0.5; 2, 5) Fo = 1; 3, 6) Fo = 10; b = $(d\bar{T}/dFo)/(d\bar{V}/dFo)$.

From the total set of similarity criteria and parameters affecting the convective drying process we sort out the most significant principal criteria and parameters. Then for the temperature coefficient b of drying we write the general expression

$$b = f(\text{Fo}, \text{Bi}_q, \text{Ly}, \varepsilon, \text{Ko}, \text{Pn}, K_1K_2).$$

The influence of the principal similarity criteria on the coefficient b is analyzed in two intervals of variation of Bi_q (from 0.1 to 1, and from 2 to 50).

The numerical values of the other criteria and parameters used in the solution are selected on the basis of the experimental data in such a way that their combined set will qualitatively reflect the drying of "thin" and "thick" materials in the declining-rate period. Their values as a function of the investigated parameter are summarized in Table 1.

We have solved the system of equations (8) and (9) on a computer to obtain the dependence of the temperature coefficient of drying on the heat- and mass-transfer similarity criteria in the indicated intervals of variation of Bi_q.

We consider the influence of the individual heat- and mass-transfer similarity criteria on the temperature coefficient of drying. The surface heat- and mass-transfer similarity criteria Bi_q and Bi_m have a strong effect on the behavior of the drying process. For small values of the Biot numbers the warmup and drying rates are insignificant, and the temperature and potential gradients of the material are small. With an increase in the Biot numbers the heat- and mass-transfer process is intensified, while at the same time the transfer potential gradients increase in the material. The heat-transfer Biot number, on the other hand, affects only the kinetics of heat transmission (T, dT/dFo) and not the mass transport (U, dU/dFo), whereas the mass-transfer Biot number acts only on the mass-transport kinetics. The numerical values of the criteria in real drying situations are approximately the same, as postulated in the solution of the equations.

An analysis of Fig. 1a shows that as the number Bi_q is increased from 0.1 to 1 the temperature coefficient of drying decreases for all values of Fo.

With a further increase in Bi_q the value of the extremal point for b changes as a function of the number Fo; for example, it corresponds to $Bi_q \approx 2$ in the case Fo = 10.

Experimental data on the drying kinetics of "thin" and "thick" samples corroborate the analytical results, although complete correspondence is not observed for Bi_q, obviously due to the simplified scheme of solution of the system of equations (8)-(9). Over the entire range of variation of Bi_q the warmup rate (dT/dFo) leads the drying rate (dU/dFo), most strikingly for large values of Bi_q.

The dependence of the temperature coefficient b of drying on the parameter K_2 is given in Fig. 1b for Bi_q = 0.5 and 10. It is seen that the temperature coefficient b increases with the parameter K_2 for any value of Bi_q. The difference lies in the intensity of the

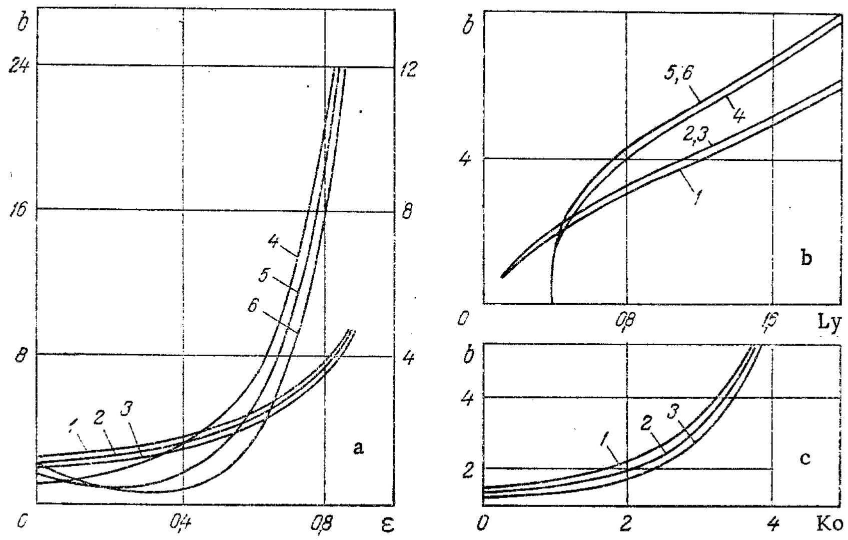

Fig. 2. Temperature coefficient of drying versus the phase-conversion criterion ε (a), the criterion Ly (b), and the criterion Ko (c). 1, 4) Fo = 0.5; 2, 5) Fo = 1; 3, 6) Fo = 10.

effect on the heat-transfer process for large values of Bi_q and K_2, as well as in the change of sequence with respect to the influence of the criterion Fo for $Bi_q = 10$.

The heat- and mass-transfer process is affected considerably by the phase-conversion criterion ε. It is apparent from Fig. 2a that the temperature coefficient of drying increases with ε for various values of Bi_q. This fact is attributable to the increase in the temperature of the material as ε is increased. For $Bi_q = 10$ (Fig. 2a, curves 1-3), however, the growth of b is far more rapid than for $Bi_q = 0.5$. This result indicates that with an increase in Bi_q the warmup rate (dT/dFo) leads the drying rate (dU/dFo) considerably. The influence of the dimensionless time Fo is significant only for $Bi_q = 10$. These analytical results are also in good agreement with experimental results on the drying kinetics of "thin" and "thick" materials.

The Kossovich (Ko) and Posnov (Pn) numbers resemble the numbers Bi_q and Bi_m in their specific attributes, but they differ from them insofar as they characterize the internal heat- and mass-transfer processes and the bound substance. The criterion Ko essentially influences the temperature fields, and the criterion Pn the moisture-content fields. It follows from Fig. 2c that the temperature coefficient b of drying increases with the value of Ko, i.e., the warmup rate leads the drying rate.

A comparison of the curves for ε and Ko as functions of the temperature coefficient b shows that the criterion Ko exerts the same influence as the criterion ε on the heat- and mass-transfer process.

In various analytical solutions one often encounters the product ε•Ko•Pn, which has a strong influence on the distribution of the moisture content in the body. The nature of the dependence of this group on the temperature coefficient of drying is naturally similar to the curves ε = f(b) and Ko = f(b). The heat and mass transfer are most affected by the Lykov number (Ly), which characterizes the inertia of the temperature field relative to the moisture-content field. The temperature coefficients of drying (Fig. 2b) increases with the value of Ly for all values of the number Bi_q. For $Bi_q = 10$ (see Fig. 2b) the growth of the coefficient b is strongest, i.e., with an increase in Bi_q the influence on the heat transfer (dT/dFo) is intensified. The influence of the criterion Fo is insignificant in this case. With an increase in the criterion Ly the warmup and drying processes are intensified for practically the same value of the dimensionless time Fo.

Consequently, it is advisable from the technological point of view to create conditions for drying such that the values of Ly will be as large as possible.

Thus, the results of the foregoing analytical investigation are qualitatively consistent with the experimental data on the drying of "thin" and "thick" materials, permitting the heat-transfer Biot number and the temperature coefficient of drying (or the Rebinder number) to be used in analyzing the drying of materials of various thicknesses.

NOTATION

$X = x/R$, dimensionless coordinate; T, dimensionless temperature; U, dimensionless moisture content; Γ, a constant coefficient ($\Gamma = 0$ for a plate); ε, phase-conversion criterion; Rb, Rebinder number; b, temperature coefficient of drying; Ko, Kossovich number; Pn, Posnov number; Ly, Lykov number; Fo, Fourier number; Ki_m, mass-transfer Kirpichev number; Bi_q, Bi_m, heat- and mass-transfer Biot numbers; μ, roots of characteristic equation; ν, characteristic numbers; $d\bar{u}/d\tau$, drying rate; N, rate of drying in the initial period; ϰ, relative drying (moisture-uptake) coefficient; R, characteristic dimension of body; R_V, volume-to-surface ratio of body; τ, time; K_1, K_2, dimensionless parameters of the drying process; a_m, mass diffusion coefficient.

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